THESIS

HIGH ACCURACY DISPLACEMENT ESTIMATION USING SONOELASTICITY IMAGING SYSTEM

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High Accuracy Small Displacement Estimation Using Sonoelastic Imaging System

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1. **General Introduction.**

Since several years after the World War II, around 50,000 to 60,000 people died from cancer every year. The number of the cancer deaths has increased steadily and cancer became the top cause of death, surpassing strokes, in 1981 according to the statistics compiled by the Ministry of Health, Labor and Welfare Japan shown in Figure 1.1.

![Trend in Mortality Rate](image)

**Figure 1.1** Mortality rate due to different type of diseases in past 50 years.

In the present cancer mortality has become a major cause of the death among the Japanese. The major cause of the number of cancer death in Japan is confronted by the increasing ratio of the aged people. For females, nearly 40% of the total deaths in the late thirties are caused by cancer and the percentage of the death increase by 10%
for late forties and late fifties respectively. Moreover in case of females, breast cancer has been increasing tremendously year by year in this present decade. This shows that the cancer attacks at the prime of the life. The number of the elderly people increases so the number of the cancer patients is also thought to be increased proportionally in the near future.

A more reliable and a sophisticated diagnosis has become a valuable need in the present context to minimize the breast cancer mortality. Though some methods of diagnosis are being used by doctors in the present time, these methods seem to be incomplete, unreliable and have a lot of side effects. Mammogram, Clinical breast exam (CBE), Magnetic resonance imaging (MRI), nuclear magnetic resonance imaging (NMRI), Tissue sampling are some methods of diagnosis that are being used in these present days but it has not yet been proven in clinical trials that use of these tests will decrease the risk of dying from cancer. Cancer screening trials are meant to show whether early detection decreases a person’s chance of dying from the disease. For some types of cancer, the chance of recovery is better if the disease is found and treated at an early stage. The present popular method, mammography, has the problem of low accuracy, side effect due to radiation and painful diagnosis. The high accuracy and reliable method is very important for the early detection of cancer having no side effect. For that reason, a number of researches are being done at different countries to establish safe and quantitative diagnosis of cancer by inspecting different parameters of viscoelastic tissue characteristics considering the fact that the cancerous tissue is harder than the normal tissue.

When a low frequency vibration of around hundred hertz to one kilohertz is applied to the soft biological tissue surface, the shear wave is generated inside the soft tissue and it propagates in the direction of vibrating wave. The shear wave propagation velocity and attenuation coefficient of the propagating wave inside the soft tissue are the important parameter for viscoelasticity measurement. The viscoelastic property of the biological tissue is closely related to the hardness and the softness of the tissue. If the viscoelastic property of the shear wave inside the tissue could be measured with high
accuracy, early detection and quantitative assessment of progression and characterization of the cancer is possible. However the mechanical structure of the biological tissue is very complex due to which uniform propagation of the wave is not possible because of multiple reflections, refractions and the absorptions of the wave. In spite of the non-uniformity and complex boundaries a very reliable method is needed to measure the viscoelastic parameters with high accuracy. Therefore in this paper different method to measure the elastic shear wave inside the soft tissue is proposed and the result is compared with the experimental result obtained from phantom. Moreover, simulation software is developed for analyzing and developing a sophisticated signal processing algorithm.
2. Research Objective.

This chapter in the first section, concerns about the importance of cancer diagnosis, problems faced in diagnosing cancer and the research objective. In the second section, we discuss on the application of low frequency vibration on soft tissue. In the third section, the introduction of the simulation program and its assumptions for deriving sophisticated signal processing algorithm is discussed.

2.1 Research Introduction.

Tissue elasticity measurement by an ultrasonic (US) wave is a promising technique to safely diagnose the condition of the breast tumor and the liver diseases. Measurement of the shear wave characteristics of the living tissue such as velocity and absorption is considered to give valuable information about tissue mechanical characteristics such as stiffness and macro structure of the tissue. Basically tissue elasticity measurement is done by two ways. One is a method in which the static pressure is used to measure the strain of the tissue but this method is not reliable because the unknown in-vivo force distribution degrades the accuracy in quantifying the in-vivo stiffness of the tissue, though the relative stiffness of the soft tissue can be obtained with high resolution. The other method is to measure the shear wave velocity in the soft tissue. It is expected that the continuous wave excitation can easily give wide range velocity information.

There are a lot of methods which are deviated for diagnosis of breast tumor in the present days. Different companies like Hitachi Medical, Siemens, and Echosens uses various excitation methods for tumor diagnosis. The Table no.2.1.1 shows the different methods followed by different companies for measuring relative stiffness and shear wave velocity. Also the table illustrates the measuring parameters with two main algorithms by applying different excitation techniques.
The Table no.2.1.1 shows that, to measure the relative stiffness of the tumor, frame to frame cross correlation is applied but the problem of quantification and repeatability caused by manual compression are the major problems. In another method of shear wave velocity measurement, time of flight is taken into concerned but due to inhomogeneity of the tissue, random scattering and multiple reflections and refractions degrades the accuracy of estimation. We should consider not only the accuracy but also the spatial resolution to obtain the stiffness and shape of a breast tumor in the diagnosis. The Japanese Foundation for Cancer reported in 2008 that a ten year survival rate of

<table>
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<th>Excitation Parameter</th>
<th>Measurement Parameter</th>
<th>Algorithm</th>
<th>Disadvantage</th>
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<tr>
<td>Real Time Elastography (Hitachi Medical systems)</td>
<td>Manual compression</td>
<td>Relative stiffness</td>
<td>Frame-to-frame cross correlation</td>
</tr>
<tr>
<td>eSie Touch (Siemens)</td>
<td>Acoustic radiation force impulse</td>
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<td>Virtual Touch Tissue Imaging (Siemens)</td>
<td></td>
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<td>Low applicability in inhomogeneous tissues due to reflection and scattering with shear wave propagation</td>
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<td>Fibroscan (Echosens)</td>
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<tr>
<td>Virtual Touch Tissue Quantification(Siemens)</td>
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93.1% is attained for an invasive breast cancer with the size of less than 10 mm. As a medical diagnosis requirement, the estimation error and the spatial resolution are expected to be within 10% and 5 mm, respectively for breast tissue. But those are not necessarily achieved in the conventional systems because multiple reflections and complex 3D propagation of the shear wave in the soft tissue greatly affect on the accuracy of the velocity estimation. Therefore the three most important terms for tissue elasticity imaging requires spatial resolution, quantification and safety. Currently the Acoustic Radiation Force Impulse (ARFI) method is widely used but this technique still have problem concerning about safety and temperature rise of the tissue as well as quantification.

We have already proposed a Low Frequency Vibration Wave (LFVW) method for safety diagnosis which excites a continuous sinusoidal wave from outside the body surface. Moreover we expected that the LFVW method has more advantage than the conventional ARFI because a pure sinusoidal wave is excited from the surface of the body. The 2D and 3D tissue displacement image is obtained by using some displacement estimation methods of the received IQ signals in a tissue Doppler measurement. In this method the 2D velocity image is successfully obtained by applying the 2D Fourier Transformation of the 2D displacement map of the of the shear wave propagation.

Generally there are numerous scatterers for US wave in soft tissue. A shear wave excited by the low-frequency vibration on the body surface makes the scatterers fluctuate. The Doppler effect of the fluctuation causes a frequency change for a US wave irradiated to the tissue. Thus the local shear wave velocity can be calculated from the local wavelength, which is estimated by the tissue displacement distribution in the shear wave propagation. Figure 2.1.2 shows a basic idea of the velocity estimation with a Virtual Sensing Array (VSA) in a soft tissue. Here the term 1D Virtual Sensing Array (VSA) is the multi-points measurement of the tissue displacement in the direction of the ultrasonic beam propagation. The simultaneous displacement measurement using multiple ultrasonic transducers gives a complete set of data acquired by 2D or 3D VSA.
A shear wave source placed at the origin vibrates with a low frequency $f$. The z-component tissue displacement $\xi_z(t, p)$ at position vector $p(x, z)$ could be expressed with wave number vector $k' = (k_x', k_z')$. Since the Doppler shift of the received US wave depends only on $\xi_z(t, p)$, an IQ signal can be obtained by convolution integral of the point spread function (PSF) defined with a shifted position vector. From the received IQ signal, a sophisticated displacement estimation method is applied to obtain high accuracy displacement data. Once the displacement is estimated, we can hence estimate the wave number vector and finally velocity by $v = \frac{2\pi f}{|k'|}$.

**Figure 2.1.2** Concept of shear wave velocity estimation using VSA.
According to the **Figure 2.1.2** shown, the low frequency wave is generated by wave source generator. US wave transducer array with elements is mechanically scanned in order to acquire the US Doppler signals in three dimensions. Center frequency, the burst length and PRF of US wave are 5MHz, 4 wavelength and 10 kHz, respectively. The IQ signal of around of 1.4MHz bandwidth is received
2.2 Low Frequency Vibration Application on soft tissue.

This section illustrates the theoretical view of the propagation of the low frequency vibration in the soft tissue inside the body. The relationship between the viscoelastic parameters and velocity of the shear wave due to low frequency vibration and the attenuation on the living tissue is derived below.

When a low frequency vibration is applied from outside the surface of the soft tissue, the longitudinal wave and the transverse wave propagates inside the viscoelastic medium and thus holds a Hooke’s law due to the fluctuation of the system. The complete system represents a Voigt model. So the propagation velocity and the attenuation constant can be calculated as shown in the following equation.

1) For Longitudinal wave:

   Propagation Velocity : \( v_l = \frac{\omega_i}{\text{Re}[g]} \) \hspace{1cm} (2-2-1)

   Attenuation Constant : \( \alpha_l = -\text{Im}[g] \) \hspace{1cm} (2-2-2)

   Where, \( g = \left( \frac{\rho \omega_i^2}{(2\mu + \lambda)} \right)^{\frac{1}{2}} \) \hspace{1cm} (2-2-3)

2) For Transverse Wave:

   Propagation velocity : \( v_t = \frac{\omega_i}{\text{Re}[h]} \) \hspace{1cm} (2-2-4)

   Attenuation Constant : \( \alpha_t = -\text{Im}[h] \) \hspace{1cm} (2-2-5)

   Where, \( h = \left( \frac{\rho \omega_i^2}{\mu} \right)^{\frac{1}{2}} \) \hspace{1cm} (2-2-6)
Here: $\mu = \mu_1 + j \omega \mu_2 \quad \lambda = \lambda_1 + j \omega \lambda_2$

Where:

- $\mu_1$: Displacement modulus of elasticity
- $\lambda_1$: Bulk modulus
- $\mu_2$: Coefficient of shear viscosity
- $\lambda_2$: Coefficient of bulk viscosity
- $\rho$: Density of the medium
- $\omega$: Angular frequency of oscillation

$\text{Re}[]$ and $\text{Im}[]$: Real and Imaginary Part of the complex term.

Although the vibrating wave inside the living tissue has both the transverse wave and longitudinal wave, but the shear wave velocity propagation is suitable for estimating the velocity of the vibrational wave inside the living tissue. The longitudinal wave is compressive and travels by compressing the medium whereas the transverse wave or the shear wave is incompressible and propagates by deforming the medium in the lateral direction equally with the velocity almost equal to the velocity of the longitudinal wave. He in this research, the energy of the vibration frequency wave is assumed to be transferred almost all to the propagating shear wave.

Considering only the transverse wave the viscoelastic parameters, propagation velocity and the attenuation of the propagating shear wave is given by the equation (2-2-4), (2-2-5) and (2-2-6) as shown below.

\[
v_t = \sqrt{\frac{2\mu_1^2 + \omega^2 \mu_2}{\rho \mu_1 + \sqrt{\mu_1^2 + \omega^2 \mu_2^2}}} \quad (2-2-7)
\]

And

\[
\alpha_t = \sqrt{\frac{\rho \omega^2 \left( \sqrt{\mu_1^2 + \omega^2 \mu_2^2} - \mu_1 \right)}{2 \left( \mu_1^2 + \omega^2 \mu_2^2 \right)}} \quad (2-2-8)
\]
For viscous elastic medium, if the modulus of elasticity of the medium is greater than coefficient of the viscosity, i.e. \( \mu_1 \gg \omega \mu_2 \), then the equation (2-2-7) and (2-2-8) will be modified to:

\[
\begin{align*}
\nu_{1i} & \equiv \frac{\mu_1}{\rho} \quad (2-2-9) \\
\alpha_{1i} & \equiv 0 \quad (2-2-10)
\end{align*}
\]

Approximately, the propagation velocity of the shear wave is related to the elastic modulus of the medium and the density of the medium. The relation shows that the propagation velocity of the shear wave will be high for the denser medium i.e. if the medium is hard and hence lesser for the rarer medium.

On the other hand, if the coefficient of viscosity of the medium is greater than the elasticity of the medium i.e. \( \mu_1 \ll \omega \mu_2 \) then the equation (2-2-7) and (2-2-8) will be modified to:

\[
\begin{align*}
\nu_{2i} & \equiv \frac{2\omega \mu_2}{\rho} \quad (2-2-11) \\
\alpha_{2i} & \equiv \sqrt{\frac{\rho \omega}{2\mu_2}} \quad (2-2-12)
\end{align*}
\]

In this case the propagation velocity and the attenuation will be frequency dependent.
2.3 Introduction of Simulator for Developing Sophisticated Signal Processing Algorithm.

In order to develop a sophisticated signal processing algorithm and to check the proposed algorithms we developed a champion signal processing numerical simulator. The simulator is designed in such a way that it covers all the components of the real time system. The velocity estimation error depends on the vibration frequency, propagation constant, additive system noise and the reflected waves from the surrounding reflector. To analyze the velocity estimation error in the various conditions described above, numerical simulations are carried out for the generation of RF signals from the fluctuated scatterers, estimation of the displacement of excitation of a plane shear wave, and evaluation of velocity estimation accuracy as a 2D problem.

The soft tissue is modeled as N numbers of ultrasonic point scatterers, which are aligned with the averaged separation of quarter wavelength of the US wave. The number of the scatterers per square of the wavelength of the US wave is approximated to some reasonable numbers. An effect of the multiple shear waves is also taken into account. The complex reflection coefficient of the scatterers has uniformly random distribution in the amplitude and the phase. The scatterers’ position fluctuates with the propagation of the plane shear wave at the applied frequency of vibration. Since the attenuation of the shear wave is nearly equals to zero, in this simulation the attenuation of the shear wave is neglected. Also we assume the reflected shear wave to have independent amplitude and the propagation direction.

The center frequency and the burst length of the transmitting US pulse are 5MHz and 4, respectively. The velocity and the attenuation constant of the US wave are assumed to be 1500 m/s and 1dB MHz\(^{-1}\)cm\(^{-1}\), respectively. For simplicity, we assume that the directivity of the US transducer depends on only the x-direction. The simulation uses transducer radius of around 2.5mm to around 5mm.
To simulate theoretical shear wave propagation in an arbitrary velocity distribution, we employ a FDTD method. In this method only the x and z direction displacement components are considered for longitudinal and shear wave. The calculation size once declared, the shear wave propagation is taken within the selected region called the Range of Interest (ROI). Also this simulation method has a facility of selecting number of layers as well as the type of medium (rectangular and circular) having specified velocity.

A complete simulation model is constructed involving the above described sections. By collecting the signals at all transducer positions, a 2D cross-sectional Doppler signal is obtained in x and z direction respectively. The complex 2D displacement distribution is estimated by the 2D cross-sectional Doppler signal. The displacement estimation methods like arc-tangent method, RF-correlation method and QI correlation method are used for estimating the displacement of the shear wave. A wave number vector filtering is applied to a small ROI for wave number calculation and hence velocity could be estimated. The estimated velocity and the velocity declared in FDTD method could be computed for analyzing the velocity estimation error. The flowchart in the Figure 2.1.3 shows illustrates the general simulation model for estimating velocity of the shear wave.
Figure 2.1.3 Simulation model flowchart.

Create Scatterers Model

Move TR position in x direction

Extract Scatterers within US beam width.

Instantaneous displacement By FDTD method.

2D Doppler signal.

IQ detection of RF signal

Displacement Estimation Methods

Arc-Tangent Method

RF Correlation Method

QI Correlation Method

Wave Number Vector for Velocity Estimation
Accuracy, calculation time, spatial resolution and resemblance with the real
time system is the most important things to be considered while making a simulator for
developing a sophisticated signal processing algorithm. The conventional signal
processing simulator consumes a lot of time for simulating a large number of data. So a
very fast simulator for estimating shear wave velocity is designed and implemented.
The details about this simulator and the comparison and the evaluation of the results
obtained from this simulator are discussed briefly in chapter 6.

In this section the basic principles of measurement of pulse Doppler signal for vibration propagation in tissue using ultrasonic wave is discussed. In second section the basic principle of shear wave propagation velocity estimation is discussed. And finally in the third section the general introduction of wavenumber filtering is discussed.

3.1 Basic Principle for Pulsed Doppler Signal Measurement.

When a low frequency vibration is applied on the surface of the living body, the vibrating wave propagates inside the body. The shear wave of the propagating vibration wave inside the body makes the small particles called the scatterers, which is less than micron meter are fluctuated from their original position. The displacement of the scatterers is measured with high range resolution by ultrasonic wave (5MHz) Doppler measurement system.

If we think of a large number of ultrasonic scattering within the soft tissue, the transmitted ultrasonic signal is modulated by the fluctuating ultrasonic scattering present inside the soft tissue. The received US signal due to Doppler Effect is thus focused on the change in frequency of the transmitted US signal. Therefore the ultrasonic wave reflected from the ultrasonic scattering can be obtained by quadrature detection of the Doppler signal estimated from the vibration propagation inside the body.
The Figure 3.1.1 shows the model of ultrasonic scattering in which the scatterers are displaced at frequency \( f_v \). The Ultrasonic transceiver is at L distance from the concerned scatterer operating at frequency \( f_0 \).

![Figure 3.1.1 Measurement model.](image)

The scatterer displacement \( \xi(t) \) is given by:

\[
\xi(t) = \xi_0 \cos \left\{ 2\pi f_0 t - \varphi \right\}
\]

(3-1-1)

Where

- \( \xi_0 \): Amplitude of oscillation of the scatterer
- \( \varphi \): Initial phase of the scatterer

If an ultrasonic pulse of frequency \( f_0 \) is transmitted from the ultrasonic transducer, the received RF signal, reflected from the scatterer is given by:
\[ y(t, \tau) = a \exp(j2\pi f_0 \tau - k_u z') \]  
\[ = a \exp\{j2\pi(f_0 + \Delta f)\tau - k_u z'\} \]  
\[ = a \exp(j2\pi f_0 \tau + \Delta \phi - k_u z') \]

Where,

\[ a \]: Amplitude of the received signal.  
\[ f_0 \]: Centre frequency of Ultrasonic wave.  
\[ c \]: Velocity of ultrasonic wave.  
\[ \Delta \phi \]: Phase change  
\[ k_u \]: Wavenumber of US pulse  
\[ z' \]: Two way distance between the US transducer and the scatterer.

The phase change of the displaced scatterer when the scatterer is displaced by \( \xi(t) \) is given by:

\[ \Delta \phi = -2 \frac{2\pi \xi(t)}{\lambda} = - \frac{4\pi f_0}{c} \xi(t) \] (3-1-3)

Where,

\[ \lambda \]: Wavelength of the Ultrasonic Wave.

On substituting the value of \( \Delta \phi \) from equation (3-1-3) on equation (3-1-2) the received RF signal will be:

\[ y(t, \tau) = a \exp\{j2\pi f_0 (\tau - 2 \frac{\xi(t)}{c}) - k_u z'\} \] (3-1-4)

If the ultrasonic frequency component of the received RF signal is convoluted and passed through the low pass filter, the Doppler signal is obtained. The Doppler signal is represented as:

\[ r(t) = \int y(t, \tau) \exp(-2\pi f_0 \tau) d\tau \]
\[ = \int a \exp\{j2\pi f_0 (\tau - 2 \frac{\xi(t)}{c}) - k_u z\} \exp(-j2\pi f_0 \tau) d\tau \]
The equation (3-1-5) shows that the Doppler shift of the received US wave depends only on the displacement component $\xi(t)$ of the scatterers.

\[
\begin{align*}
\exp \left( - j \frac{4 \pi f_0}{c} \xi(t) \right) & = a \exp \left( - j \frac{4 \pi f_0}{c} \xi_0 \cos \{2 \pi f_0 t - \varphi\} \right) \\
\end{align*}
\]
3.2 Basic Principle for Shear Wave Propagation Velocity Estimation.

Let us consider a vibrator operated at low frequency, vibrates the tissue from the outer surface of the body. The Figure 3.2.1 shows the general model of the system. The ultrasonic transducer transmits and receives the ultrasonic wave to and from the soft tissue. The received signal is then processed to estimate Doppler signal due to the displaced scatterer on the soft tissue.

![Figure 3.2.1 General model of the system](image)

It is assumed that the soft tissue is characterized by the N number of very small like particles called the scatterers. If we limit the Range of Interest (ROI) to small square area, the displacement in the ROI can be modeled as superposition of the small number of plane waves. Let us consider a vibrational source placed at the origin then the plane shear wave between the very small intervals of the scatterers donot have reflection and refraction. Also we consider a closed surface area represented by \((X,Y,Z)\) coordinate of VSA width \(d\) in all direction, which accommodate N number of scatterers. Then the propagation angle of each wave in z direction is \(\theta_{ij}\).
(i = 1, 2, 3,.....N), (l = 1, 2, 3.....L_i) for a shear wave of frequency $f_i$ and $L_i$ wave number. The general model of the system shown in Figure 3.2.1 is calibrated with the above specified divisions and its lateral and top views are shown in Figure 3.2.2 and Figure 3.2.3 respectively.

**Figure 3.2.2** Lateral view of the selected crosssectional area of scatterer.
If we consider a first shear wave of frequency $f_i$, the respective wavenumber vector in x, z and y direction will be represented by $k'_{i,tx}$, $k'_{i,tz}$ and $k'_{i,ty}$ respectively. And their respective values are:

$$k'_{i,tx} = k_i \sin \theta_{i,t} \sin \phi_{i,t}$$
$$k'_{i,tz} = k_i \cos \theta_{i,t}$$
$$k'_{i,ty} = k_i \sin \theta_{i,t} \cos \phi_{i,t}$$

From the the equation (3-2-1) the three dimensional displacement amplitude due to vibrational shear wave is given by the equation (3-2-2) shown below.

$$\xi(p,t) = \sum_{i=1}^{N} \sum_{l=1}^{L_i} \delta_{i,l} \cos \left\{ 2\pi f_i t - (k'_{i,tx}x + k'_{i,tz}z + k'_{i,ty}y + \phi_{i,t}) \right\}$$

$$= \sum_{i=1}^{N} \sum_{l=1}^{L_i} \delta_{i,l} \cos \left\{ 2\pi f_i t - (k'_{i,t} \cdot p + \phi_{i,t}) \right\}$$

Where

$f_i$ : Excitation Frequency of $i^{th}$ shear wave

$\delta_{i,l}$ : Angle between the $i^{th}$

$k = (k'_{i,tx}, k'_{i,tz}, k'_{i,ty})$ : Wavenumber vector of the shear wave.
\[ p = (x, z, y) \quad : \text{Position vector} \]

\[ \phi_{i,j} \quad : \text{Initial phase} \]

The complex amplitude \( \hat{\xi}(p, f_i) \) can be obtained by performing the Fourier Transform of the displacement amplitude due to vibrational frequency within the measurement time interval \( T \). The complex amplitude is then given by:

\[
\hat{\xi}(p, f_i) = \frac{2}{T} \int_0^T \xi(p, t) \exp(-j2\pi f_i t) dt
\]

\[
= \frac{2}{T} \int_0^T \sum_{i=1}^{N} \sum_{j=1}^{L_i} \delta_{ij} \cos\left\{2\pi f_i t - (k'_{ij} \cdot p + \phi_{ij})\right\} \exp(j2\pi f_i t) dt
\]

\[
= \sum_{i=1}^{L_i} \delta_{i} \exp\{-j(k'_{ij} \cdot p + \phi_{ij})\}
\] (3-2-3)

If the position vector \( P(X, Y, Z) \) is the center position of the VSA aperture, then the wave number spectrum of is given by the 3D Fourier Transformation of the complex displacement amplitude \( \hat{\xi}(p, f_i) \) as shown below in equation (3-2-4).

\[
P(k, P, f_i) = \frac{1}{d^3} \int_{-d/2}^{+d/2} \int_{-d/2}^{+d/2} \int_{-d/2}^{+d/2} \hat{\xi}(p, f_i) \exp\{-j(k \cdot p)\} dx dy dz
\]

\[
= \sum_{i=1}^{L_i} \delta_{i} \exp\left\{j\{k \cdot k'_{ij}\}\right\} \sin c\left\{\frac{k_x - k'_{ij}}{2}d\right\} \sin c\left\{\frac{k_z - k'_{ij}}{2}d\right\} \sin c\left\{\frac{k_y - k'_{ij}}{2}d\right\}
\] (3-2-4)

Where,

\( d \) : Aperture width.
The wave-number vector $k'_i$, of the elastic shear wave can be estimated by searching the local peak position of the spectrum of the wave-number vector $k'_i$. Therefore the velocity of the shear wave can be calculated as:

$$v(P, f_i, l) = \frac{2\pi f_i}{|k'_i|} \quad (3-2-5)$$

The azimuth angle is given by

$$\phi(P, f_i, l) = \tan^{-1}\left(\frac{k_x}{k_y}\right)$$

And the angle from top view is given by

$$\theta(P, f_i, l) = \tan^{-1}\left(\frac{\sqrt{k_y^2 + k_x^2}}{k_z}\right)$$
3.3 Basic Principle of Wavenumber Filtering.

Not only is the propagation velocity of the shear wave, the propagation direction of the shear wave is also a useful parameter. For this reason the wavenumber filtering is selected as the sophisticated algorithm for shear wave propagation direction estimation. Even for the multiple shear waves such as reflected and refracted shear waves propagating inside the soft tissue, the propagation direction could be estimated by selecting the 1\textsuperscript{st}, 2\textsuperscript{nd} or 3\textsuperscript{rd} peak of the wavenumber spectrum. Hence, filtering in the wave number domain allows us to obtain the propagation direction dependency of the visco-elastic properties. The wave number Filtering is applied to the small ROI of the measured complex displacement image. The wave number filtering is applied to the small ROI of the measured complex displacement image. The filtered spectrum is obtained after filtering by wavenumber filter.

In this chapter, basic principle of displacement estimation method is discussed in detail. The displacement is estimated from the in-phase I (real part) and quadrature Q (imaginary part) signal received from the quadrature detector. The following steps shows the general flow of the shear wave velocity estimation and the step at which the displacement estimation is done from the QI signal obtained from complex Doppler signal.

- Excitation of shear wave inside the living body using low frequency vibration.
- US wave is transmitted and reflected US pulse received and is used for Doppler signal measurement.
- Quadrature detection is done and Complex Doppler signal is estimated.
- Displacement methods applied to estimate vibration displacement.
  - Arc-Tangent Method.
  - RF correlation Method.
  - QI correlation Method
- Estimate velocity and its direction by using wavenumber filtering.

The above described research flow shows that the three different displacement estimation methods use the IQ signal calculated from the complex Doppler signal for displacement estimation. The basic formulation and displacement estimation techniques of all three methods are discussed briefly in the following sections.
4.1 Arc-Tangent Method.

The arc-tangent method is one of the promising methods in the signal processing for computing the principle value of the argument function applied to the complex number in which the argument can be changed by $2\pi$ (especially from $-\pi$ to $+\pi$) without making any difference to the angle. The arc-tangent method is applied to complex IQ signal for estimating very small displacement inside the tissue. The complex Doppler signal illustrated in the equation (3-1-5) is again rewritten in the equation (4-1-1).

$$r(t) = a \exp\{-j \frac{4\pi f_0}{c} \xi \cos\{2\pi f_c t - \phi\}\}$$

(4-1-1)

If we consider the z-direction component of the complex Doppler signal the arc-tangent method is applied, the small z-direction displacement is given by the equation (4-1-2).

$$\xi_z(t) = -\frac{c}{4\pi f_0} \left\{ \tan^{-1}\left(\frac{\text{Im}[r_z(t)]}{\text{Re}[r_z(t)]}\right) \pm 2n\pi \right\}$$

(4-1-2)

Where,

- $\text{Im}[r_z(t)]$: Imaginary part or the quadrature Q signal of the complex Doppler signal.
- $\text{Re}[r_z(t)]$: Real part or the in phase I signal of the complex Doppler signal.
- $\pm 2n\pi$: Offset estimated from zero-crossing point of $\text{Re}[r_z(t)]$.

It is known that the measuring accuracy using the quadrature detector is decreased under the proper conditions of the received signals. To improve the accuracy of the calculation, theoretical correction of the error due to the random structure of the tissue was applied.
4.2 RF correlation Method.

Based on the fact that the received RF signal could be reconstructed from QI signal and vice versa, the RF cross-correlation computation is proposed. The simulation flowchart shown below in the Figure 4.2.1 shows the basic procedure for high accuracy small displacement estimation.

Figure 4.2.1 Simulation flowchart.
The above flowchart in Figure 4.2.1 gives information about from where the RF signal could be obtained. In practice the received frequency modulated (FM) signal obtained from ultrasonic scanning is passed through the quadrature detector which splits the signal into I and Q signal with 90° phase shift. In general the RF (Radio Frequency) signal received from the hardware is represented by the equation (4-2-1) and its IQ signal will be represented by equation (4-2-2).

\[ y(t) = a(t) \sin(\omega_0 t + \phi(t)) \]  
\[ \text{(4-2-1)} \]

Where,
\[ y(t) : \text{Received RF Signal.} \]
\[ a(t) : \text{Amplitude of the RF Signal at time’s’.} \]
\[ \omega_0 : \text{Center frequency of the Ultrasonic wave.} \]

The IQ signal derived from the equation (4-2-1) is given by,

\[ I(t) = a(t) \sin \phi(t) \]
\[ \text{And} \]
\[ Q(t) = a(t) \cos \phi(t) \]  
\[ \text{(4-2-2)} \]

The Figure 4.2.2 shows the part of system by which the QI signal is obtained.
In order to calculate cross-correlation of the RF signal, first of all the RF signal should be reconstructed from the IQ signal. The term RF in ultrasound industry, which is the output of the beamformer, is a standard notation for the unprocessed data, where the frequency information is intact. The real valued RF signal is multiplied (“mixed”) with two complex sinusoid signals of 90° phase difference. This process is called the down mixing. After down mixing the complex signal is filtered to remove the negative frequency spectrum and noise outside the desired bandwidth. Finally IQ demodulation is done by decimating the filtered signal. The RF reconstruction procedure is briefly explained in the next section 4.2.1.

\[ y(t) = a(t) \sin(\omega_0 t + \phi(t)) \]
4.2.1 Reconstruction of RF Signal from IQ signal.

The QI demodulation preserves the information content in the Band-pass signal, and the original RF signal can be reconstructed from the IQ signal. The reconstruction of the RF signal is straightforward. It is a reversal of the complex demodulation process. The decimation is reversed by interpolation. The low pass filter cannot be reversed, but it can be chosen without the loss of information. The down mixing is reversed by up mixing and finally the RF signal is found by taking the real-value of the complex up-mixed signal. Up mixing in this case is achieved by just multiplying the interpolated IQ signal by the inverse of the complex exponential which was used when down mixing. The reconstruction of RF signal is well illustrated by the flowchart shown in Figure 4.2.1.1.

![Complex RF signal Reconstruction diagram.](image)

**Figure 4.2.1.1** Complex RF signal Reconstruction diagram.
The RF reconstruction block diagram in **Figure 4.2.1.1** shows that the three necessary steps are followed before up-mixing of the IQ signal in-order to preserve the loss of information. The three steps are briefly discussed below.

**Zero Padding:** Zero padding means inserting zeros in between the signals to increase sampling rate. In frequency domain the added zeros will be seen as number of new replicas of the low-pass spectrum, spaced with the original sampling frequency.

**Up sampling:** In this section the received IQ signal is up sampled. The purpose of up-sampling is to decimate the IQ signal for up-mixing.

**Low Pass Filtering:** After insertion of zeros, the duplicate spectra should be removed. This is done with low pass filtering, most probably the FIR –filter with sinc coefficients.

Mathematically, the RF signal reconstruction is illustrated in the equations below. Let us consider the IQ signal from the **equation (4-2-2-1)**. The IQ signal when up mixing is done i.e. if I and Q signal is multiplied with cosine and sine components respectively at the ultrasonic center frequency and summed up; the real part of the RF signal is obtained.

\[ I(t) = a(t) \sin \phi(t) \]  
**equation (4-2-2-1)**

And

\[ Q(t) = a(t) \cos \phi(t) \]  
**equation (4-2-2-2)**

After up-mixing,

\[ Q(t) \sin \omega_0 t = a(t) \cos \phi(t) \sin \omega_0 t \]

\[ = \frac{1}{2} a(t) [\sin(\omega_0 t + \phi(t)) + \sin(\omega_0 t - \phi(t))] \]  
**equation (4-2-2-3)**
Also,

\[ I(t) \cos \omega_s t = a(t) \sin \phi(t) \cos \omega_s t \]
\[ = \frac{1}{2} a(t) [\sin(\omega_s t + \phi(t)) - \sin(\omega_s t - \phi(t))] \quad (4-2-4) \]

From **equation (4-2-2-3)** and **equation (4-2-2-4)**, the real part of the RF signal is obtained by summing up Q and I signal.

\[ RF_{Re}(t) = Q(t) \sin \omega_s t + I(t) \cos \omega_s t \]
\[ = a(t) \sin(\omega_s t + \phi(t)) \quad (4-2-5) \]

Similarly,

\[ I(t) \cos \omega_s t = a(t) \cos \phi(t) \cos \omega_s t \]
\[ = \frac{1}{2} a(t) [\cos(\omega_s t - \phi(t)) + \cos(\omega_s t + \phi(t))] \quad (4-2-6) \]

Also

\[ I(t) \sin \omega_s t = a(t) \sin \phi(t) \sin \omega_s t \]
\[ = \frac{1}{2} a(t) [\cos(\omega_s t - \phi(t)) - \cos(\omega_s t + \phi(t))] \quad (4-2-7) \]

From **equation (4-2-2-6)** and **equation (4-2-2-7)**, the imaginary part of the RF signal is obtained by summing up Q and I signal.

\[ RF_{Im}(t) = Q(t) \cos \omega_s t + I(t) \sin \omega_s t \]
\[ = a(t) \cos(\omega_s t + \phi(t)) \quad (4-2-8) \]
Finally the reconstructed complex RF signal is given by,

$$\tilde{RF}(t) = RF_{Re} + RF_{Im}$$
$$= a(t) \sin(\omega_R t + \phi(t)) + a(t) \cos(\omega_R t + \phi(t))$$
$$= a(t) \exp(\omega_R t + \phi(t)) \tag{4-2-9}$$

The reconstructed complex RF signal resembles with the received RF signal achieved from the output of the beamformer. The reconstructed RF signal is used for calculating cross-correlation, which is described in section 4.2.2. And this parameter is used for estimating the small displacement.
4.2.2 Small displacement estimation from RF correlation.

In this section, calculation of cross-correlation of the reconstructed RF signal is done by shifting the reconstructed RF signal by a unit pulse of the US wave. The reconstructed RF signal and the reconstructed time shifted RF signal are used for estimating the small displacement. The flowchart in the Figure 4.2.2.1 shows the general procedure followed to estimate the small displacement of the RF signal.

![Flowchart for displacement estimation from Reconstructed RF signal.](image)
The flowchart in Figure 4.2.2.1 shows that the square error between the up-sampled reconstructed RF signal and the up-sampled reconstructed time shifted RF signal is calculated in order to calculate the normalized signal power of the RF signal. The square error normalized by the signal power is then used for estimating the minimum square error displacement by the least square fitting method. Mathematical derivation of the small displacement is shown in the following equations.

The initial reconstructed RF signal is given by,

\[ RF(t) = y(t) = a(t) \exp(-j\omega t + \phi(t)) \]  \hspace{1cm} (4-2-2-1)

If the RF signal is shifted by one pulse (i.e. by time \( \Delta t \)) the phase will be shifted by angle \( \Delta \phi = 2\pi f_0 \Delta t \) as shown in the Figure 4.2.2.2.

![Figure 4.2.2.2 Initial and time shifted RF signal.](image_url)

The time shifted, reconstructed RF signal is then given by the equation (4-2-2-2).

\[ RF(t + \Delta t) = y(t + \Delta t) = a(t + \Delta t) \exp(-j\omega_0 (t + \Delta t) + \phi(t + \Delta)) \]  \hspace{1cm} (4-2-2-2)

Therefore the correlation coefficient of the RF signal is given by,

\[ RF(\Delta t) = RF(t) \cdot RF(t + \Delta t) \]
If the RF correlation is calculated for the total time of flight $\tau$ in the $z$-direction, then the correlation coefficient will be given by the equation (4-2-2-3).

$$RF_\tau^z(\Delta t) = \int_0^\tau RF(\Delta t)dt$$

$$= \int_0^\tau RF(t).RF(t + \Delta t)dt$$

(4-2-2-3)

The normalization factor represented by $e_{norm}(\Delta t)$ can be calculated as,

$$e_{norm}(\Delta t) = RF_\tau^z(\Delta t) / \int_0^\tau RF^2(t)dt$$

(4-2-2-4)

By using the least square fitting method, the small displacement with minimum square error is finally estimated. In this least square fitting method, the correlation coefficient data is fitted with the curve defined by $y = ax^2 + bx + c$ where the variable $x$ with minimum $y$ is estimated as $-b/2a$. Here the estimated value of $y$ is the estimated displacement. The fitting curve is shown in the Figure 4.2.2.3.

![Figure 4.2.2.3 Fitting curve for Least Square Fitting Method.](image-url)
4.3 QI correlation Method.

In this section, small displacement estimation is made on the basis of Q, I signal and time shifted Q, I signal separately. By this procedure, it is expected that the QI correlation method reduces the CPU execution time for small displacement estimation. The Q and I signal are separately time shifted and the correlation coefficient is calculated. The small displacement is estimated by least square fitting error method which is same as explained in RF correlation estimation method. For this let us consider the real part of the RF signal which is represented by equation (4-3-1).

\[RF_{Re}(t) = Q(t)\sin\omega_t + I(t)\cos\omega_t\]  
(4-3-1)

If the RF signal is shifted by time \(\Delta t\) is shifted by one pulse, then the time shifted RF signal is given by the equation (4-3-2).

\[RF_{Re}(t+\Delta t) = Q(t+\Delta t)\sin\omega_0(t+\Delta t) + I(t+\Delta t)\cos\omega_0(t+\Delta t)\]  
(4-3-2)

From equation (4-3-1) and equation (4-3-2), the correlation coefficient of the two real valued IQ signal is then given by,

\[RF_\tau(\Delta t) = \int_0^\tau RF_{Re}(t)RF_{Re}(t+\Delta t)dt\]

\[= \int_0^\tau [(Q(t)\sin\omega_0(t) + I(t)\cos\omega_0(t))]\]
\[\times[(Q(t+\Delta t)\sin\omega_0(t+\Delta t) + I(t+\Delta t)\cos\omega_0(t+\Delta t))]dt\]

\[= \frac{1}{2}\int_0^\tau [(Q(t)Q(t+\Delta t) + I(t)I(t+\Delta t)\cos\omega_0(t+\Delta t))]
\[+[(I(t)Q(t+\Delta t) - Q(t)I(t+\Delta t))\sin\omega_0\Delta t]]dt\]  
(4-3-4)

The normalization factor represented by \(e_{\text{norm}}(\Delta t)\) can be calculated as,

\[e_{\text{norm}}(\Delta t) = RF_\tau(\Delta t) / \int_0^\tau RF^2(t)dt\]  
(4-2-2-4)
By using the least square fitting method, the small displacement with minimum square error is finally estimated. In this least square fitting method, the correlation coefficient data is fitted with the curve defined by $y = ax^2 + bx + c$ where the variable $x$ with minimum $y$ is estimated as $-b/2a$. 
5. Comparison and Evaluation of proposed displacement Estimation methods.

In this chapter, the validity of the proposed RF correlation method and the QI correlation method is compared with the conventional Arc-Tangent method. The assumptions taken in all three methods and their demerits are discussed briefly. The evaluations of these three methods are done by taking the QI data from the experimental data performed on Agar-phantom with different specification. The first section in this chapter deals with the developed measurement system. The second section deals with the comparison among the three methods. The second section discusses about the assumptions, merits and demerits of all these three methods and their results are compared using the agar-phantom experimental data. Finally in the last section the execution time taken by three methods are discussed.

5.1 Developed Measurement System.

To demonstrate the validity of the proposed method, experiments were carried out with a phantom. The Figure 5.1.1 shows a block diagram of the developed measurement system. A variable sinusoidal signal of different vibrational frequency can be generated by a function generator and is fed to the vibrator (Emic 512-A, maximum displacement of 7mm, exciting force 49 to 64 N) through a power amplifier up to 90 Vpp according to the desired requirement. A shear wave is excited into a phantom by a plastic sphere having a diameter of 15mm.

A US pulse having the center frequency and burst length of 5 MHz and 4, respectively, is generated by a field programmable gate array (FPGA)-based pulse generator. The pulse repetition frequency is 10 kHz, which is more than 10 times larger than the vibration frequency. The circular planner US transducer having the diameter of 5 mm is driven with the 40 Vpp US pulses. The receiving time gate is determined between 10 and 40 mm in depth. After quadrature detection, the received Doppler signal is analog-to-digital (AD)-converted in 5 MHz sampling for 20ms. This system achieves
the dynamic range of 70 dB in the Doppler components at the vibration frequency when the measurement time is 20ms. The US transducer scans over the agar phantom with the interval of 0.25mm with a computer controlled 2D positioner. Since the trigger timing is synchronized between the vibration signal and the US pulse sequence, displacement caused by vibration is measured two dimensionally inside the phantom.

Figure 5.1.1 Block diagram of developed measurement system.
5.2 Comparison of Displacement Estimation Methods.

In this section the drawbacks of the conventional methods, the assumptions made on proposed correlation methods are discussed. Moreover the validity of the proposed methods is compared with the conventional arc-tangent method and it is done by using the experimental IQ data received from the experimental setup.

Assumptions:

5.2.1 Arc-Tangent Method.

In case of the Arc-tangent method, the displacement estimation is done using Q and I signal in each point of the z direction. Therefore the displacement value in each point of z-direction is truly dependent on the individual value of Q and I signal. This can be well defined by the equation (5-2-1-1) of Doppler signal and the equation (5-2-1-2) of Arc-tangent.

\[
 r(t) = a \exp\{-j \frac{4\pi f_0}{c} \xi_0 \cos\{2\pi f_c t - \varphi\}\} \tag{5-2-1-1}
\]

On applying arc-tangent method,

\[
 \xi_z(t) = -\frac{c}{4\pi f_0} \left\{ \tan^{-1}\left( \frac{\text{Im}[r_z(t)]}{\text{Re}[r_z(t)]} \right) \pm 2n\pi \right\} \tag{5-2-1-3}
\]

With respect to the equation (5-2-1-3), if the real value i.e. I signal of the complex Doppler signal becomes zero then the error due to zero division degrades the dominant displacement estimation and hence produces spike like noises.

5.2.2 RF correlation Method.

In order to minimize the division by zero problems of arc tangent method, a RF correlation method is proposed. In this method the correlation coefficient and the fitting error is calculated and displacement is estimated in a specified volume. The least square fitting method minimizes the displacement error by normalizing the fitting error within
the total pulses to average value.

Another assumption made on RF correlation method is that the correlation coefficient is taken within the sample volume displaced uniformly in the z-direction.

5.2.3 QI correlation Method.

QI correlation method is also applied under the same condition as that of RF correlation method. Only the difference is that, in this method, the correlation coefficient is calculated by shifting the Q and I signal separately. This is illustrated by the equation (5-2-3-1).

\[
RF_c(\Delta t) = \frac{1}{2} \int_0^{T_0} \{ [Q(t)Q(t + \Delta t) + I(t)I(t + \Delta t) \cos \omega_0(t + \Delta t)] \\
+ [(I(t)Q(t + \Delta t) - Q(t)I(t + \Delta t)) \sin \omega_0 \Delta t] \} dt
\]  

(5-2-3-1)

5.3 Experimental Evaluations and Verifications.

In order to compare the results of the three displacement estimation methods, the QI data from the agar-phantom experiment is taken and simulated for displacement estimation. For this purpose the agar-phantom which has the elastic property same as the biological tissue is prepared by mixing the graphite with agar. The preparation method is described below.

- Predetermined amount of Graphite and agar powder is added to the measured amount of boiling water.
- It is then stirred and is cooled up to 40°C so as not to precipitate graphite at the bottom.
- The mixture is then put in the refrigerator for some hours.
  (For example: to prepare 1% agar-phantom, we use agar of 1.5% of volume of water (always fixed) and graphite of 1% of volume of water (changes according to the percentage of agar-phantom).
The experimental comparison is done with two types of agar-phantom. The experimental set up for both the experiments are same and it is explained in section 5.1. Only the phantom specification is different. To excite the vibration shear wave inside the phantom, the plastic sphere attached with the vibrator is made in contact at the spherical top surface of the phantom. The internal displacement of the phantom is then measured by the US transducer scanning perpendicularly on the vertical direction.

The experiment is performed on the agar-phantom of diameter 20 cm and depth 13 cm, in which the ultrasound scanning is done 15 mm far from the excitation point. The Figure 5.3.1 and the Figure 5.3.2 shows the experiment state and the overview of the experiment respectively.

![Figure 5.3.1. Agar Phantom.](image)

![Figure 5.3.2 Experiment overview](image)
5.3.1 One layered Phantom.

The specifications for experiment are:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phantom Type</td>
<td>1% agar-phantom (single Layered)</td>
</tr>
<tr>
<td>Measurement time (t)</td>
<td>20 msec.</td>
</tr>
<tr>
<td>Measurement area:</td>
<td>at x=10 mm in x-direction, for total measurement length of 25mm in x-direction. 30mm in z-direction</td>
</tr>
<tr>
<td>Vibrational frequency</td>
<td>500Hz</td>
</tr>
<tr>
<td>Ultrasound Central Frequency</td>
<td>5MHz</td>
</tr>
<tr>
<td>Ultrasonic scan interval</td>
<td>0.25mm</td>
</tr>
<tr>
<td>Vibration Source</td>
<td>Plastic sphere of 15mm diameter</td>
</tr>
</tbody>
</table>

The Figures below shows the 2D displacement plot of the phantom due to shear wave excitation inside the phantom for all three displacement estimation methods.

**Figure 5.3.1.1** Displacement Plot by Arc-tangent Method.
Figure 5.3.1.2 Displacement Plot by RF correlation Method.

Figure 5.3.1.3 Displacement Plot by QI correlation Method.
5.3.2 Two layered Phantom.

The specifications for experiment are:

Phantom Type 1.5% agar-phantom (Upper layer depth of 2.5cm)  
and 1% agar-phantom (Lower layer depth of 10 cm)

Measurement time (t): 20 msec.

Measurement area: at x=80 mm in x-direction, for total measurement

length of 20 mm in x-direction, 30mm in z-direction

Vibrational frequency 750Hz

Ultrasound Central Frequency 5MHz

Ultrasonic scan interval 0.25mm

Vibration Source Plastic sphere of 15mm diameter

The figure below shows the 2D displacement plot of the two layered phantom 
due to shear wave excitation inside the phantom for all three displacement estimation methods.

![Figure 5.3.2.1 Experiment overview](image-url)
The figure below shows the 2D displacement plot of the two layered phantom for all three displacement estimation methods.

**Figure 5.3.2.2** Displacement Plot by Arc-tangent Method.

**Figure 5.3.2.3** Displacement Plot by RF correlation Method.
Figure 5.3.2.4 Displacement Plot by QI correlation Method.

The plots in the section 5.3.1 and section 5.3.2 shows the displacement plot of single layered phantom at the distance of 2.5 mm far from the vibrating source and in case of double layered phantom at the distance of 20 mm far from the vibrating source. The red marked pins show that the errors are present in the same position of the displacement plot figure. The bending of the displacement plot seen in the blue oval markings in each figure also illustrates the unusual behavior of the received signal.

The presence of the error in all displacement estimation method shows that the error is due to the received signal but not due to the error evolved by the displacement estimation method. The plot of individual signal at each point shows that the presence of the error is due to the decrease in amplitude of the received signal at that point.
The 2D amplitude plot and phase plot as well as the velocity mapping estimated by the three different displacement estimation methods are also analyzed for checking the validity and CPU execution time. In this case the **correlation width is 1.5 \( \mu m \)**

- For **Single layered Phantom** at vibrational frequency of **500Hz**.

(1.a) Arc-tangent

(1.b) RF Correlation

(1.c) QI Correlation

(2.a) Arc-tangent

(2.b) RF Correlation
Figure 5.3.2.5 Displacement phase plot (fig. 1.a, 1.b, and 1.c), Displacement Amplitude plot (fig. 2.a, 2.b, and 2.c) and Velocity mapping (fig. 3.a, 3.b, and 3.c) for Arc-Tangent (fig. a), RF Correlation (fig. b) and QI Correlation (fig. c) Methods respectively.
For **Double layered Phantom** at vibrational frequency of **750Hz**.

(4.a) Arc-tangent

(4.b) RF Correlation

(4.c) QI Correlation

(5.a) Arc-tangent

(5.b) RF Correlation
Figure 5.3.2.6 Displacement phase plot (fig. 4.a, 4.b, and 4.c), Displacement Amplitude plot (fig. 5.a, 5.b, and 5.c) and Velocity mapping (fig. 6.a, 6.b, and 6.c) for Arc-Tangent (fig. a), RF Correlation (fig. b) and QI Correlation (fig. c) Methods respectively.
The displacement amplitude plot, displacement phase plot and the velocity plot shown in Figure 5.3.2.5 and Figure 5.3.2.6 for the single layered phantom and the double layered phantom shows that result is almost same in case of all three algorithms.

In case of the RF correlation method and QI correlation method the precision value is very high so that a very clear phase map could be plotted. In RF correlation method it is very easy to locate the noise or the error signal. Thus the advantage of analyzing the error present or the noise present on the system could be done to develop a more sophisticated signal processing algorithm.
5.4 Execution Time Evaluation.

The computational time for shear wave velocity estimation is also an important factor for quick diagnosis of the tumor. For this reason the scanning of transducer as well as the small displacement estimation and velocity estimation should be time limited. In this section the discussion is made on the basis of displacement computational time of all the three displacement estimation methods.

5.4.1 Arc-Tangent Method.

The RF signal received by the US transceiver is converted into I Q signal by quadrature demodulator. The Arc-Tangent method, described by the equation (5-4-1-1) directly computes on the received Q and I signal for each pulse. Therefore the displacement estimation is relatively fast than the RF correlation method and QI correlation method. The Figure 5.4.1 shows the general flowchart for displacement estimation by Arc-Tangent method.

\[ \xi(t) = -\frac{c}{4\pi f_0} \tan^{-1} \left( \frac{I(t)}{Q(t)} \right) \]  

(5-4-1-1)

![Flowchart for Arc-Tangent Method](image.png)
5.4.2 RF Correlation Method.

To increase the accuracy of the displacement estimation and to figure out the error position, the RF correlation method is selected for estimating the displacement. The RF signals reconstructed from the Q and I signal is decimated (up-to 1GHz) and the correlation coefficient is calculated for RF signals and the time lagged RF signals for each sampling period. The displacement estimated for each sampling interval and the least square fitting method applied to each pulse consumes high execution time. Due to this reason, the execution time for displacement estimation method is extremely high with compared to the Arc-Tangent method. The time consumption section during displacement estimation is shown in Figure 5.4.2.

![RF Correlation Method flowcharts.](image)}
5.4.3 Q I Correlation Method.

It is optimized that if I and Q signal is directly used for calculating the correlation coefficient, the execution time for displacement estimation will be minimized. The correlation coefficient calculated between I and Q signal and time lagged I and Q signal without decimation eliminates the execution time spent in calculating cross correlation coefficient at each sampling interval. Only some of the time will be spend on least square fitting of the estimated displacement. This will result on reduction of execution time than the time taken by RF correlation method, even though it is little bit slower than the Arc-Tangent method. The time consumption section is shown in the Figure 5.4.3.

\[
r(\Delta t) = \frac{1}{2} \left\{ (Q(t)Q_1(t+\Delta t) - J(t)I_1(t + \Delta t)) \cos \omega_c \Delta t \right\} \\
+ \left\{ (I(t)Q_1(t + \Delta t) - Q(t)I_1(t + \Delta t)) \sin \omega_c \Delta t \right\}
\]

\( \Delta t \): time lag

**Figure 5.4.3** RF Correlation Method flowcharts.

In this section the conventional velocity estimation simulator along with its assumptions are discussed. The conventional velocity estimation simulator consumes a long time if the velocity estimation error or the different parameter are needed to be evaluated. In order to minimize the CPU calculation time a new algorithm has been proposed with certain assumptions. The second section in this chapter discusses about the assumptions made in the proposed algorithm and the concrete flowchart of the proposed algorithm. Finally in the last section, the results from the proposed algorithm are compared with conventional algorithm and the Agar-phantom to check the validity of the proposed algorithm.

6.1 Conventional Velocity Estimation Simulator.

To analyze the velocity estimation error in various conditions and to select a sophisticated signal processing algorithm, a numerical simulation is carried out. The simulator is designed in such a way that the assumptions made are near to the actual value. A numerical simulation has been carried out to generate RF signals from fluctuated scatterers, estimation of displacement of a plane shear wave and evaluation of velocity estimation. The soft tissue is modeled as the numerous point scatterers, which are randomly aligned with the averaged separation of a quarter wavelength of the US wave. The number of scatterers per wavelength of the US wave is approximated about 150. The scatterer position fluctuates with the propagation of the plane shear wave at a frequency of some hundred hertz. The attenuation of the shear wave is neglected and the reflected shear wave is assumed to have independent amplitude and propagation direction. To simulate theoretical shear wave propagation in an arbitrary velocity distribution, FDTD (Finite-Difference Time-Domain) method is employed.

The center frequency and the burst length of the transmitting US pulse are 5MHz and 4 respectively. The velocity and the attenuation constant of the US wave are assumed to be $1500\text{m/s}$ and $1\text{dB MHz}^{-1}\ \text{cm}^{-1}$, respectively.
In this simulation, the directivity of the US transducer depends on only the x-direction. When the two way path length between the transducer and the scatterer is determined, a pulse response convolved by the US pulse is calculated in at a high sampling frequency (1GHz). The received RF signal is generated by accumulating all the pulse responses from the scatterers the base band signals are decimated by 10 MHz and calculated at every pulse repetition period of 0.1 ms in 20 ms for Doppler analysis.

Collecting the signals at all transducer positions, a 2D cross-sectional Doppler signals are obtained with every 0.25mm and 0.75e\(^{-6}\) m (75 \(\mu\)m) in x and z directions respectively. The complex displacement distribution is estimated by the 2D cross-sectional Doppler signal and finally the velocity of the shear wave is evaluated.

The concrete algorithm flowchart for the conventional simulator is shown in the Figure 6.1.1.
6.1.1 Drawback of the Conventional Velocity Estimation Simulator.

According to the flowchart, the pulse response of the scatterer due to two way path length between the scatterer and the US transducer is convolved by the US pulse at very high frequency. This is because the phase change of the scatterer due to change in position of the transducer is effective in case the wavelength of the propagating shear wave is nearly equals to beam width of the ultrasound wave. Therefore for each scatterer, the convolution between the impulse response of the scatterer and the US pulse should be performed in order to obtain the received RF signal. Moreover the convolution is performed at very high sampling frequency of 1GHz; the CPU execution time is tremendously increased.

6.2 Proposed Velocity Estimation Simulator.

In order to overcome the drawback of the conventional velocity estimation algorithm, a new concept is proposed to reduce the CPU execution time of the simulator. For this, a large number of scatterers are created inside the beam of the US transducer position and the beam width is divided into very small divisions in which the beam division width is very less than the wavelength of the propagating shear wave. Therefore the change in phase of the scatterer due to change in beam divisions within the position of the US transducer is constant for all beam divisions. So, once the phase shift of the scatterers within the beam division width is calculated, it can be used as the constant phase shift for all the scatterers within the transducer position. This procedure will continue in each transducer position.

On applying the above mentioned concept, the CPU execution time will tremendously reduce by 10 times of the execution time taken by conventional velocity estimation simulator. The Figure 6.2.1 illustrates the concrete algorithm for the proposed velocity estimation simulator.
Figure 6.2.1 Flowchart for Proposed Algorithm.
6.3 Comparison and Verification.

The result obtained from the proposed algorithm is compared with the result of conventional algorithm as well as with the result of phantom experiment for its verification.

6.3.1 Comparison with Conventional Algorithm.

The phase plot for the displacement estimation achieved from the proposed simulator is compared with the phase plot achieved from proposed simulator. The shear wave propagation by an arbitrary velocity distribution is modeled by the Finite-Discrete Time-Division (FDTD) method. The simulation is performed with the specification (parameters) shown in the Figure 6.3.1.1.

**FDTD Model**
- Shear wave Velocity : 3 m/s
- Vibration Frequency : 500Hz
- Range of Interest(ROI) : 20mm in x & z
- Type of Medium : Homogeneous
- Number of grid =301
- Grid size = 0.1mm

![FDTD simulation model](image)

**Figure 6.3.1.1** FDTD simulation model and Range of Interest (ROI) parameter.
For the Ultrasound frequency of 5 MHz, the phase plot of the estimated displacement is plotted in two sections of up to 15 mm in depth (z-direction) and below 20 mm in depth (z-direction) by using Arc-Tangent method. The estimated phase plot for both the conventional and proposed algorithms are shown in Figure 6.3.1.2 and Figure 6.3.1.3 respectively.

**Figure 6.3.1.2** Fig (2.a) Phase plot up to 15mm by Conventional Algorithm

**Figure 6.3.1.2** Fig (2.b) Phase plot up to 15mm by Proposed Algorithm
Fig (3.a) Phase plot from 20mm to 35mm by Conventional Algorithm

Figure 6.3.1.3 Fig (3.b) Phase plot from 20mm to 35mm by Proposed Algorithm

The phase plots in both the cases show that the wavelengths in both cases are same for displacement estimation. The mapping of phase plot for proposed algorithm below 20mm in depth is more matched with the conventional algorithm than the upper portion. This is because in the proposed algorithm for simplicity the directivity of the
ultrasonic transducer is assumed to be unity i.e. the directivity is independent of the measuring depth in z-direction.

**6.3.2 Comparison with Phantom Experiment.**

The displacement phase plot of the single layered agar-phantom (1%) is also compared with the displacement phase plot of the proposed simulation algorithm under the same experimental condition. The phase plots are shown in Figure 6.3.2.1.

**Fig (3.a)** Phase plot in 15mm × 15mm area on Phantom Experiment data.

**Figure 6.3.2.1 Fig (3.a)** Phase plot in 15mm × 15mm area by Proposed Algorithm.
The phase plot shown in Figure 6.3.2.1 shows that the experimental data almost resemble with the result of the proposed algorithm. This proves the validity of the proposed algorithm. The main objective of this proposed algorithm is to minimize the computational time of the simulation. And this purpose has been successfully achieved where the execution time of the proposed algorithm has been reduced by 20 times than the conventional algorithm. The measured time graph of the conventional and the proposed algorithm has been shown in Figure 6.3.2.2.

Figure 6.3.2.2 The execution time plot between Conventional and Proposed Algorithm.
7. Conclusion.

In order to develop a highly sophisticated system for imaging the tumor, simulator, hardware as well as the high accuracy estimation algorithms is equally important. The thesis concerns mainly on two major topics, one is development of displacement estimation method and the other is high speed simulator for simulating large number of database.

The developed RF correlation displacement estimation method has the high precision estimation through which a very high quality phase mapping of the small displacement could be done. Through this correlation method, noise position could be located in the imaging by selecting individual signal waves. Though the RF correlation method is time consuming it could be a gateway for characterizing noise nature. Moreover the proposed QI correlation method is optimized to decrease the execution time than RF correlation method, increasing the precision of estimation than the conventional Arc-Tangent method.

The proposed simulator is also very fast to solve very large number of databases. It also serves as a good tool to calculate the estimation error of any parameter through simulation.
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References


